FSJM –FINAL- 9 MAY 2020

Information and results at http://www.fsjm.ch/

START for ALL PARTICIPANTS

1. NUMBER WHEEL (coefficient 1)

A wheel rolls without slipping on a paved road. The wheel is divided into eight segments numbered 1 to 8 and, as it rolls, a segment always rolls exactly on a paving stone.

What will be the number of the segment rolling on the 21st paving stone?

2. TWO HOUSES (coefficient 2)

Lea and Leo live in houses on the same street. Each house has a two-digit number. The difference between the two house numbers is equal to 20 and their sum is equal to 120. Lea's house has a larger number than that of Leo's.

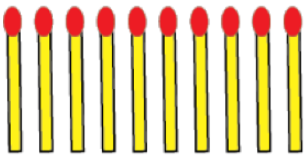
What is Lea's house number?

3. FROM 1 TO 15 (coefficient 3)

Matthew writes the numbers 1 to 15 in the boxes of this board, so that there are never two odd numbers in two boxes whose sides touch each other. He adds up all the numbers written in the grey boxes.

What is the total?

4. THE MATCHES (coefficient 4)



Matilda puts 10 matches in front of her. She offers Matthew the following game: they take turns removing 1 match, 2 matches or 3 matches. Whoever takes the last match wins. Matilda plays first.

How many matches does she take on her first turn to be sure of winning, whatever Matthew plays?

5. WHO TOOK THE ORANGE? (coefficient 5)

The shopkeeper questions four young rascals to find out who took an orange from him.

"It's Alice" said Michael.

"No, it's Frances," said Alice.

"In any case, it's not me," said Kevin.

"Alice is a liar," said Frances at last.

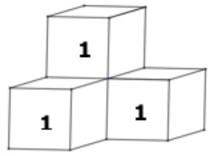
Only one child lied.

Who took the orange?

END for CE PARTICIPANTS

6. STACKS OF DICE (coefficient 6)

Mathias decides to stack four normal dice, as in the figure. Recall that on a normal die, the sum of the numbers located on opposite sides is always equal to 7. The four dice are placed on a table. The faces in contact with the table and the faces between two cubes are not visible.

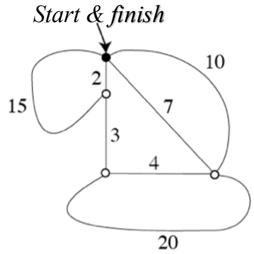


What is the sum of the visible numbers, at most?

7. WALKS IN THE FOREST (coefficient 7)

Zoe decides to go for walks in the forest near her house: this is a small sketch of the possible paths with their length in hectometres.

She wants to walk circuits without going over the same path twice during a walk, but she does not necessarily travel all the paths during an outing. During the same circuit, she can pass twice through the same intersection, including her departure point.



If she wants to walk a different distance in one walk each day, how many days will it take to exhaust all possibilities?

8. FAIR SHARES (coefficient 8)

Matthew found eight lotto game tokens, numbered 1 to 8.

He asks Matilda to separate these tokens into two groups so that the sum of the numbers in each group is the same. Matilda immediately finds a solution:

1 + 2 + 3 + 4 + 8 = 5 + 6 + 7 = 18.

The box of tokens also contains all the tokens numbered 9 to 20. Matilda then asks Matthew:

If I add one-by-one to your eight tokens the token 9, then the 10, then the 11, etc., up to 20, in order and without skipping numbers, in how many cases will I have a number of tokens where we can split into two groups of the same amount?

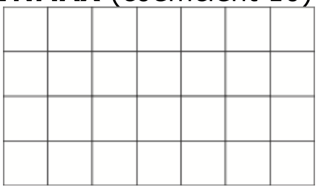
Note: We will only count cases from 9 tokens.

END for CM PARTICIPANTS

Problems 9 to 18: beware! For a problem to be completely solved, you must give both the number of solutions, AND give the solution if there is only one, or give any two correct solutions if there are more than one. For all problems that may have more than one solution, there is space for two answers on the answer sheet (but there may still be just one solution).

9. GENE'S JEANS (coefficient 9)
Gene's jeans are made of $0.6m^2$ of denim plus two holes, the total area of which is 20% of that of the denim. Each wash wears out the jeans some more, with $10cm^2$ of denim being replaced by 10 cm^2 of holes.
After how many washes will the area of holes be as much as that of denim?

10. PAWNMAX (coefficient 10)

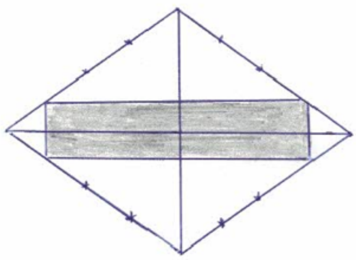


Matthew places pawns on this rectangular grid of 7 boxes by 4. He must not place pawns in 3 adjacent boxes aligned in a row, a column or diagonally.
How many pawns can he place?

11. THE MAGIC NUMBER (coefficient 11)
Matilda has found a 4-digit number, which does not contain the digit 0. The hundreds digit of this number is double that of the thousands digit, and the units digit is double that of the tens. In addition, by dividing this number by the sum of its digits, we get a whole number.
What is Matilda's number?

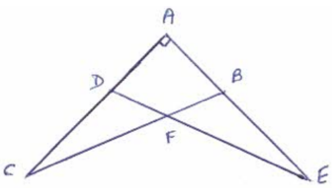
END for C1 PARTICIPANTS

12. GEOMETRIC RATIO (coefficient 12)
The sides of a rhombus are divided into quarters, and a rectangle is inscribed in it according to the drawing.



What is the ratio of the area of the rectangle to the area of the rhombus?
Give the answer to three decimal places.

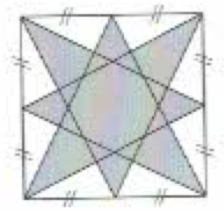
13. CONDITION OF EQUALITY (coefficient 13)
The right-angled triangles ABC and ADE are isometric. The area of the quadrilateral ADFB is equal to the sum of the areas of the triangles CFD and BFE.
What is the ratio of length AB to length AC?
Give the answer as an irreducible fraction.
Note: The figure is not to scale.



14. … AND DOUBLE 20 (coefficient 14)
Matilda wrote down a number with at most six digits. Matthew wrote 20 in front of Matilda's number and obtained a first number A. He then successively wrote a 2 then a 0 to the right of Matilda's number and thus obtained a second number B. The numbers A and B therefore both have two more digits than Matilda's number. Matilda points out to Matthew that of the two numbers A and B, one equals 2/5 of the other.
What was Matilda's initial number?

END for C2 PARTICIPANTS

15. MY STAR PART (coeff. 15)

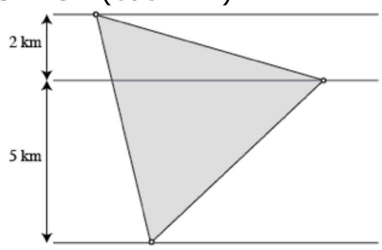


What percentage of the area of the square is the area of the star inscribed in it?
Give the answer in % to the nearest 1%.

16. TWELVE DIVISORS (coefficient 16)
The number 2020 has 12 divisors. When these divisors are written in ascending order, the 7th is a prime number.
Which future year of the 21st century will have 12 divisors, and when these divisors are arranged in ascending order, the 7th will be a prime number?
Note: there can be prime divisors before the 7th.

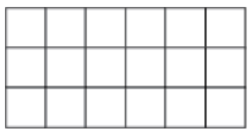
END for L1, GP PARTICIPANTS

17. THE TRIANGULAR FOREST (coef. 17)
This forest has the shape of an equilateral triangle. Three parallel roads pass respectively through each of its three corners. Two of the roads are spaced 2km apart; the third is spaced 5km and 7km from the first two (see figure).



How long is each side of the forest?
Give the answer will in km to the nearest metre.
If necessary, take 1.414 for √2; 1.732 for √3; 2.236 for √5; 3.6056 for √13.

18. SMALL AND BIGGER SQUARES (coeff. 18)
In the figure, there are 32 squares: 18 small squares and some bigger.



In a much larger rectangle also made up of small squares all of the same size, there are in total 1365 squares, whatever their size.
How many small squares does this larger rectangle have?

END for L2, HC PARTICIPANTS