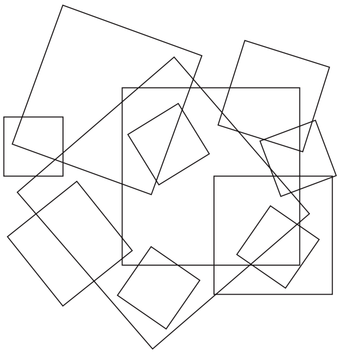
FSJM – SEMI-FINAL- 20 MARCH 2021

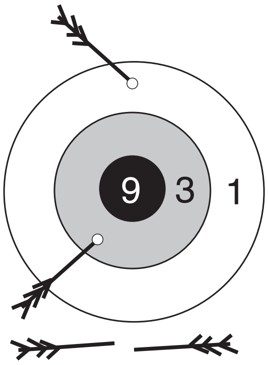
Information and results at http://www.fsjm.ch/

START for ALL PARTICIPANTS

1. Squares (coefficient 1)



Matilda has drawn eleven squares in her exercise book. She decides to erase some of them such that the sides of the remaining squares never intersect. **How many squares should she erase, at a minimum?**

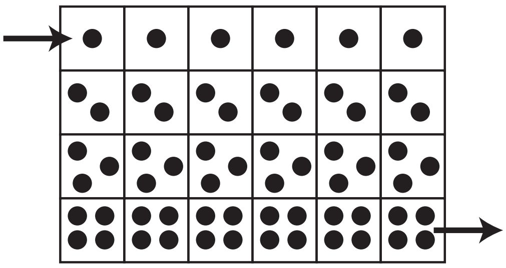


2. Darts (coefficient 2)

Matthew has a target and four darts. On the example in the drawing, he scored only 4 points as two darts fell outside the target.

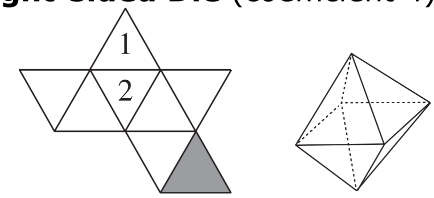
What's the smallest total he can't achieve with four darts?

3. Chambers in the Labyrinth (coeff. 3)



Each of the chambers in this labyrinth contains 1, 2, 3 or 4 gold coins as shown in the figure. Once you have entered the labyrinth (arrow at the top left), you can only move right or down. You must collect all the gold coins from the chambers you pass through. **How many different paths allow you to collect a total of exactly 21 gold coins before exiting (arrow lower right)?**

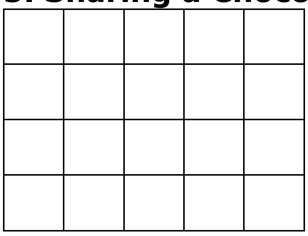
4. Eight-sided Die (coefficient 4)



The figure shows the pattern of an eight-sided die, on which you want to write the numbers 1 to 8, and the shape of this die once assembled. When assembled, the sum of the numbers on every pair of opposite sides is the same.

What number will be written on the shaded side?

5. Sharing a Chocolate Bar (coefficient 5)



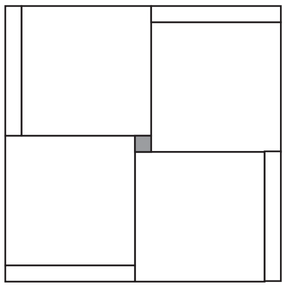
Matthew has a 4-by-5 square rectangular chocolate bar. He wants to share it with his three friends while keeping a piece for himself. Each should receive an unbroken piece and all the pieces should be the same shape and consist of whole squares of chocolate.

How many different shapes of pieces could each friend receive?

We consider rotated or upside-down pieces as the same.

END for CE PARTICIPANTS

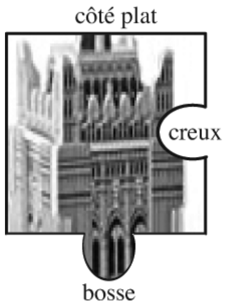
6. Ceiling Lamp (coefficient 6)



A square ceiling is covered with four identical square tiles and four identical rectangular tiles leaving only a small square space free (in grey) to hang a lamp.

Arranging the eight tiles in different ways, without overlapping, on the ceiling, how many different places are there where you can hang the lamp?

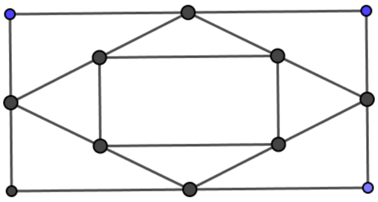
7. The ConsecYears (coefficient 7)
A ConsecYear is a year written as two consecutive whole numbers, such as 78 (from 7,8) or 2021 (from 20,21). Matilda adds up the numbers of the ConsecYears from years 12 to 2021.
What will be the result?
Note: Writing a multi-digit whole number never starts with a 0.



8. Jigsaw Puzzle Pieces (coefficient 8)
Matthew received as a gift a jigsaw puzzle with a large number of pieces which are all based on a square of the same size. The four corner pieces have two flat sides, like the example, but can also have two hollows or two bumps. The other edge pieces all have one flat side. No interior piece has a flat side, each side having a hollow or a bump. The hollows and bumps are the same size and are in the middle of the sides. To sort the pieces of his puzzle, Matthew makes piles, each comprising all those pieces that overlap perfectly, each piece being placed either right side up or upside-down.
When he has sorted all the pieces, how many piles will Matthew get, at most?

END for CM PARTICIPANTS

Problems 9 to 18: beware! For a problem to be completely solved, you must give both the number of solutions, AND give the solution if there is only one, or give any two correct solutions if there are more than one. For all problems that may have more than one solution, there is space for two answers on the answer sheet (but there may still be just one solution).



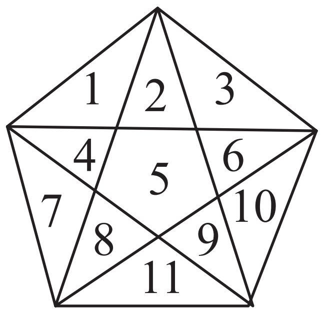
9. Even Smaller (coefficient 9)
Matilda wants to create a mosaic that gives an idea of infinity. She draws a rectangle of 8m by 4m on the ground and inscribes a diamond whose vertices are the midpoints of the sides of the rectangle, then she inscribes in this diamond a second rectangle whose vertices are the midpoints of the sides of the diamond, and so on, alternating diamonds and rectangles. She stops as soon as she inscribes a figure with an area smaller than $1 dm^2$.
How many diamonds did Matilda draw?

10. Eight Divisors (coefficient 10)
Lisa has fun adding the numbers 1 to 21 and finds that the resulting number has eight divisors.
What is the smallest number that has exactly eight divisors?

11. Gift Wrapping (coefficient 11)

Emil wants to give his girlfriend a box of chocolates of volume $225 cm^3$. The box is a rectangular parallelepiped with all edges measuring whole numbers of cm strictly greater than 1cm, with the top and bottom faces being square. He wants to put a nice ribbon around it, placed as shown in the drawing which does not necessarily respect the proportions of the box.
Knowing that the knot uses 25 cm of ribbon, what will be the overall length needed to wrap the gift?

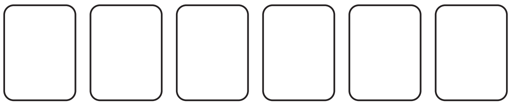
END for C1 PARTICIPANTS



12. Diagonals (coefficient 12)
John draws a regular pentagon and all its diagonals. These delimit eleven regions within the pentagon.

How many regions will he get if he draws a regular heptagon and all of its diagonals?
A heptagon is a 7-sided polygon.

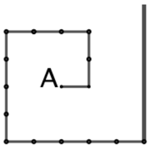
13. Play the Lottery (coefficient 13)
Alex likes to play the lottery where you have to fill in a grid of 6 numbers randomly chosen from the range 1 to 42. On the grid, the numbers will be arranged in ascending order.



He plays each time by writing 6 numbers by using the 10 digits from 0 to 9, each once and only once, such as the selection 5, 7, 18, 26, 39, 40.
How many different selections can he compose?
Note: a number never starts with a 0.

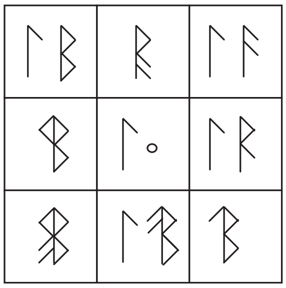
14. Fair Shares (coefficient 14)
We consider the set E = {1; 2; 3; 4; 5; 6; 7; 8}. We can divide it into two subsets E_1 = {1; 2; 3; 4; 8} and E_2 = {5; 6; 7}, such that the sum of the elements of E_1 equals the sum of the elements of E_2.
If we consider the set E of the numbers which go from 1 to *n*, where *n* can take all the values from 1 to 21, for which value of *n* can we divide the set E into two subsets E_1 and E_2, such that the sum of the elements of E_1 is equal to the sum of the elements of E_2?

END for C2 PARTICIPANTS



15. The Robot (coefficient 15)
A robot moves in a spiral on a grid as shown. He takes steps of 50 cm. Starting from point A, take one step east then turn 90 ° to the left and advance 2 steps north, then turn 90 ° and advance 3 steps west, turn 90 ° and go 4 paces south and so on. In each new direction he takes one more step than in the previous direction.
After walking 2080 steps, how far from its starting point is it?
If necessary, take 1.414 for √2 and give the answer in cm and round to the nearest cm.

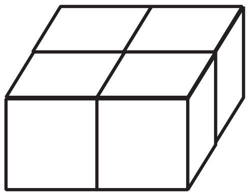
16. Kryptonian Square (coeff. 16)
A magic square has just reached us from the planet Krypton. This square contains nine integers all different written in a positional numbering system similar to ours (numbers written from left to right) but whose base is not necessarily decimal. The sum of the numbers written in each row, each column and each of the large diagonals is always the same.



By comparison with other Kryptonian texts, it is established that ° corresponds to our digit 0, that  = 9 and that  represent three consecutive whole numbers written in ascending order.
What is the sum of the nine numbers written in this square?
This sum to be given in base 10.

END for L1, GP PARTICIPANTS

17. Four Small Cubes (coefficient 17)



Adelaide paints the six faces of a 2 x 2 x 1cm rectangular parallelepiped green, then cuts it into four cubes of 1cm per side. Her little sister Sophie takes the four cubes then, without taking into account the painted faces, assembles them randomly and glues them together so as to reconstitute a parallelepiped of dimensions 2 × 2 × 1cm.
What is the probability, expressed as an irreducible fraction, that the visible area painted green on the six faces of the new parallelepiped is 12 cm²?

18. Sharing the Garden (coefficient 18)
In the town of Mathville, there is a triangular garden ABC whose sides measure AB = 36m, AC = 38 m and BC = 60 m. The gardener wishes to build a rectilinear fence DE which connects the sides AB and BC (D is located on the side AB and E on the side BC) so that the two parts of the garden thus delimited by the fence have the same area and also the same perimeter.
How many metres from B are the two ends of the barrier?

END for L2, HC PARTICIPANTS