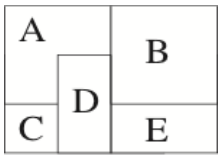
# 1st ONLINE WORLD MATH CONTEST 2021 – 28 AUGUST 2021

## START FOR ALL PARTICIPANTS

**1. IN ORDER** (coefficient 1)

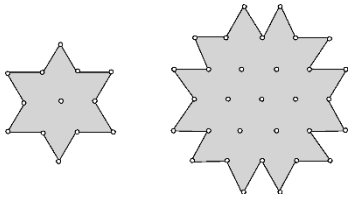



Five squares marked with the letters A, B, C, D and E, all the same size as square B, have been placed on the rectangle in a certain order.
**Which letter identifies the square that was placed third?**

**2. THE STAIRCASE** (coefficient 2)
Ethan is at the bottom of a four-step staircase.
In one movement, he is either climbing one step or climbing two steps at once.
**How many different ways can Ethan get to the top of the stairs, knowing that he always advances his left foot first?**

**3. BABA** (coefficient 3)
A letter always represents the same digit which is not 0.
The number written as AB is equal to the sum of the four digits of the number written as BABA.
**What is the number written as AB?**

**4. THE STARS & THE VIRUS** (coeff. 4)
Matilda has some stars identical to that shown on the left. Gluing several of these stars together, some of which may partially overlap, she creates the ugly virus shown on the right.



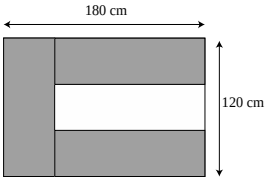
**How many stars did she use, at a minimum?**
*Note: all the small triangles formed between the white dots are identical.*

**5. THE CHIPS** (coefficient 5)
At *Casino Royale*, a machine, when you introduce tokens ("*chips*"), gives back more chips. It gives back 2 red chips for an inserted blue chip, or 3 blue chips for an inserted red chip.
At the start, James has 6 red chips but he has no blue chips.
He successively inserts 10 tokens.
In the end, James has 20 tokens in total, with at least one of each colour.
**How many blue chips does he have at the end?**

## END FOR A PARTICIPANTS

**6. SCHOOL BUS** (coefficient 6)
A school bus can hold a maximum of thirty passengers.
At the start of its route, it contains as many girls as boys.
At the first stop, half the girls get off and one boy gets on. At the second stop, a fifth of the passengers get off, then the driver starts up again.
**How many passengers are in the bus after this restart?**
*Note: the bus driver is never counted.*

**7. THE FLAG** (coefficient 7)
A country's new flag is in the form of a rectangle of dimensions 120cm x 180cm. It is divided into four small rectangles (three grey and one white) all of the same area.




**What is the length in centimetres of the small white rectangle in the drawing?**

**8. BLACK, BUT NOT TOO MUCH** (coefficient 8)
We want to build a large cube with 27 small cubes, all the same size. 12 of the small cubes are black and 15 are white.
There should be a small black square, corresponding to one face of a small black cube, in the centre of each face of the large cube.
By counting the small black squares on each side of the large cube, we get the numbers from 1 to 6.
As for a normal die used for games, the sum of the numbers obtained on any two opposite sides must be equal to 7.
**How many small black cubes will there be at the eight vertices of the large cube?**

## END FOR B PARTICIPANTS

*Problems 9 to 18: beware! For a problem to be completely solved, you must give both the number of solutions, AND give the solution if there is only one, or give any two correct solutions if there are more than one. For all problems that may have more than one solution, there is space for two answers on the answer sheet (but there may still be just one solution).*

**9. LCM** (coefficient 9)

Our two friends want to distribute an assortment of 35 chocolates. Patricia separated the 35 as: 16 + 8 + 5 + 4 + 2. The Lowest Common Multiple (LCM) of these five numbers is 80.

Patrick wants to separate the 35 into the sum of five distinct positive numbers, so that their LCM is as small as possible.

**What will this LCM be?**

**10. 2021, IN SUM** (coefficient 10)

2021 is the sum of seven distinct positive integers, every digit of which is either 3 or 5.

**In total, how many digits "3" will there be in these seven numbers?**

**11. THE TRIPLES** (coefficient 11)

We write the non-zero integers in order and without leaving any gaps between them: 123456789101112131415 ...

Then we cut the sequence thus obtained into triples each of three digits:

(123)(456)(789)(101)(112)(131)(415) ...

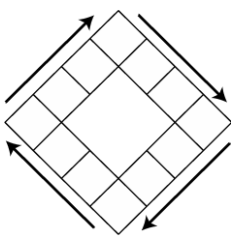
Finally, we stop as soon as the triple (101) has just been written for the fifth time.

**How many times has the trio (131) been written before that point?**

## END FOR C PARTICIPANTS

**12. SQUARE OF CUBES** (coefficient 12)

Write a number in each empty box around the edge of this square so that each four-digit number, read in the direction of the arrow corresponding to it, is the cube of a positive whole number, the first digit of each number never being a 0.



**Which of these four cubes is the smallest, or one of the smallest, if two or more are equal?**

**13. THE AQUARIUM** (coefficient 13)

An aquarium for transporting small goldfish is shaped as a rectangular parallelepiped. It holds 6 litres of water and is perfectly waterproof (1 litre = 1000 $cm^3$).

If the aquarium is placed horizontally on a certain face, the water level is 3 cm below the opposite face. If it is laid horizontally on another face, the water level is 4 cm below the opposite face. If it is laid horizontally on yet another face, the water level is 5 cm below the opposite face.

**What is the volume of the aquarium in $cm^3$?**

Give the answer rounded to the nearest $cm^3$.

**14. GRASSHOPPERS** (coefficient 14)

Non-zero positive integers, not necessarily distinct, are written on the squares of an 8 × 8 chessboard (one number per square).At the beginning, five grasshoppers are on five different squares and hide the numbers.

Gabriel calculates the sum of all the visible numbers and he obtains 100.

Simultaneously, each grasshopper jumps onto an adjacent square (it crosses a side shared by two squares).

Gabriel calculates the sum of all the visible numbers and he gets 1000.

And so on, until he can no longer obtain a sum ten times greater than the previous one (when Gabriel calculates a sum, two grasshoppers are never on the same square).

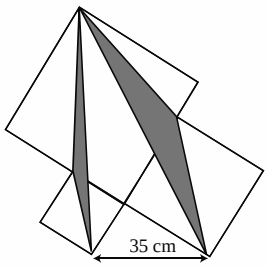
The total of the sixty-four numbers written on the chessboard is divisible by 35 and it is the largest possible.

**What is this total?**

## END FOR D PARTICIPANTS

**15. CLOCK HANDS** (coefficient 15)

A cubist painting represents part of the dial of a Swiss clock.




The three squares are adjacent and they share a vertex.
The distance between the ends of the two hands is 35 cm.

**What is the grey area in the drawing?**

*Give the answer rounded to the closest $mm^2$.*

**16. A STREET, IT ADDS UP** (coef. 16)

In this street, all the houses are numbered continuously from 1 to 1000.
Alice lives at number 6. The sum of the numbers from 1 to 5 is equal to that of the numbers from 7 to 8, i.e. 15.
Then, Bob lives at number 35. The sum of the numbers from 1 to 34 is equal to that of the numbers from 36 to 49, i.e. 595.
Finally, Cedric lives at number N, different from Alice's and Bob's numbers.
The sum of the numbers from 1 to (N - 1) is equal to that of the numbers from (N + 1) to a certain number strictly greater than (N + 1).

**What is the value of N?**

**END FOR E, F PARTICIPANTS**

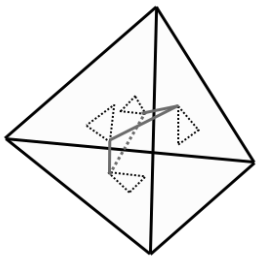
**17. STUTTERS** (coefficient 17)

Fibo plays with a series whose first term is 1 and whose second term is 1, then in which each term is the sum of the two terms that precede it:
1, 1, 2, 3, 5, 8, 13, ...
He starts from the first term, multiplies it by 10 and adds the second, multiplies the result by 10 and adds the third, and so on. Fibo thus obtains
1, 11, 112, 1123, 11235, 112358, 1123593 (= 112358 x 10 + 13), ...
After a while, he gets blocks of digits that repeat one after another to infinity.

**How many digits do these blocks contain, at a minimum?**

**18. THE TRAPPED FLY** (coef. 18)

When a fly enters a lantern, it ends up traveling a path made up of four straight segments of equal length.
The lantern is a regular tetrahedron of side 30 cm.



If we draw in the centre of each face a small equilateral triangle of a certain size, always the same, whose sides are parallel to the edges of the tetrahedron, the path bounces off that face at a vertex of the triangle.
The path is as short as possible.

**What is its length rounded to the nearest mm?**

*Note: if square roots are involved, perform the calculations by taking 1.414 as √2, 1.732 as √3 and 2.236 as √5.*

**END FOR G PARTICIPANTS**

PICTET 1805 EPFL n|w Fachhochschule Nordwestschweiz ETH zürich