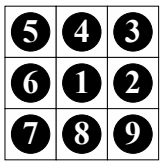
FSJM – SEMI-FINAL- 19 MARCH 2022

Information and results at http://www.fsjm.ch/

START for ALL PARTICIPANTS

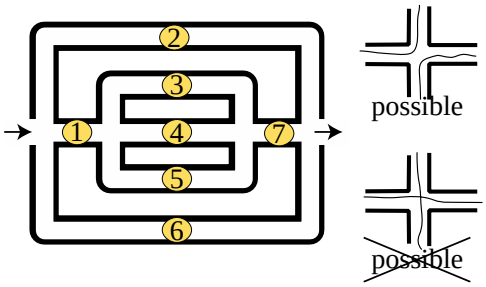
1. Nine Tokens (coefficient 1)



Nine tokens numbered from 1 to 9 are placed on a square grid as shown in the figure. Matthew removes three tokens so that there are only two tokens left in each row and in each column of the grid.
He adds the numbers of the three tokens he removed.
What will be the total, at most?

2. Ariadne's Thread (coefficient 2)

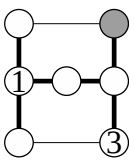
Ariane entered this maze from the left and came out of it from the right, after having collected the seven gold coins numbered from 1 to 7 in an order that you have to find. She unrolled a thread behind her that never crossed itself and she never passed through a corridor twice (but she could pass twice through a junction if she did not cross the thread, see the example above).



What are the numbers on the fourth and sixth coins she picked up?

3. From 1 to 7 (coefficient 3)

Place the numbers 2, 4, 5, 6 and 7 in the empty discs (1 and 3 are already placed) so that:



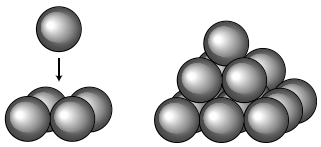
- the sum of three aligned numbers (thick lines) is always equal to 12;
- the sum of the five numbers placed around each small rectangle is equal to 20.

What number should be written in the grey disc?

4. Matilda's number (coefficient 4)

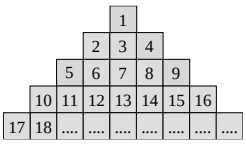
Matilda has found a two-digit positive whole number such that if we multiply its digits together, we find exactly twice the result obtained by adding them.
Her number is the smallest two-digit positive whole number with this property.
What is her number?

5. Cannonballs (coefficient 5)



Matthew arranges four identical cannonballs in a square, each ball touching its two neighbours, then he places a fifth ball on the first four. There are then a total of eight points of contact between these five balls.
Mathias then builds a pyramid in the same way, but with fourteen balls: nine balls on the first level, four on the second and one ball on the third, each ball touching all its neighbours.
How many contact points does this pyramid, shown on the right, have in total?

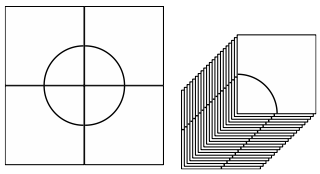
END for CE PARTICIPANTS



6. Matilda's Pyramid (coefficient 6)

Matilda has built a pyramid of consecutive whole numbers starting from 1 as shown in the drawing which represents the top five rows. Matilda's pyramid has 22 rows.
How many odd numbers does the bottom row have?

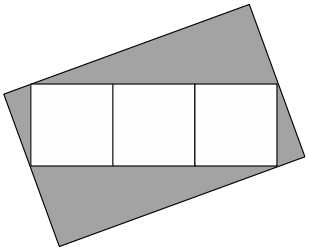
7. Matthew's Cards (coefficient 7)



Matthew has a large number of identical square cards each bearing a quarter circle that joins the midpoints of two sides of the square. By arranging four cards, he can form a circle like the one shown in the figure, but he can also form other longer curves by using more cards.

If he uses more than four cards, without folding them or overlapping them, how many cards, at least, will he need to form a closed curve?

8. 3 Squares on a Rectangle (coef. 8)



Matthew has glued three white squares onto a grey rectangle. The three squares are perfectly aligned, without overlapping. Four of the vertices of the three squares are on the edge of the grey rectangle, two of which are at the midpoints of the shorter sides of the rectangle.

If each white square has an area of 22 cm^2, what is the area of the large grey rectangle?

Give the answer in cm^2.

END for CM PARTICIPANTS

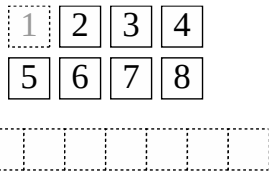
Problems 9 to 18: beware! For a problem to be completely solved, you must give both the number of solutions, and give the solution if there is only one, or give any two correct solutions if there are more than one. For all problems that may have more than one solution, there is space for two answers on the answer sheet (but there may still be just one solution).

9. Posts and Enclosures (coefficient 9)

Sylvie owns a circular plot of land of radius 20 m. She decides to form triangular enclosures of side 10 m for her horses. She places a post at each vertex of the triangles, one of these posts being at the centre of the field.

How many posts can she place, at most?

10. 8 Cards (coefficient 10)



Matilda has eight cards bearing the numbers from 1 to 8.

She places the card numbered 1, as shown in the figure, then the other seven in a row next to it so that the sum of two numbers written on adjacent cards is never divisible by 2 nor by 3.

What will be the eight-digit number on the cards when placed in a row?

11. A Total of a Year (coefficient 11)

Matilda would like to write 2022 as the sum of the numbers from 1 to 100, each preceded by a + sign or a – sign, using the fewest possible number of – signs:

$2022 = \pm 1 \pm 2 \pm 3 \pm 4 \pm \cdots \pm 99 \pm 100$.

Reading the equality from left to right, the first – sign appears as late as possible.

What will be the first number preceded by a – sign?

Give the answer as a positive number (with no – sign).

END for C1 PARTICIPANTS

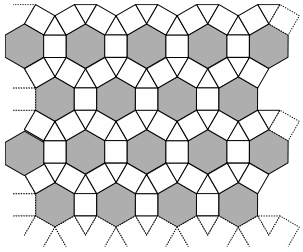
12. Lottery Tickets (coefficient 12)

Matilda and Matthew have each bought a lottery ticket.

Each ticket bears a five-digit number. The first two digits of Matthew's ticket are 09. By comparing their tickets, they note that the ten digits from 0 to 9 all appear on these tickets and that doubling the number on Matthew's ticket (without taking into account the initial 0) gives Matilda's ticket number.

What is Matilda's ticket number?

13. Diana's Paving (coefficient 13)

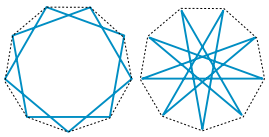


This paving, found in Nîmes, in a temple dedicated to Diana, consists of equilateral triangles, squares and regular hexagons.
It is assumed that an (unlimited) plane is entirely covered using this paving.
What is the relationship between the number of hexagons and the number of polygons (triangles, squares and hexagons) in the paving?
Give the answer in as an irreducible fraction.

14. LEON and NOEL (coefficient 14)
Leon and Noel each coded their first name with a four-digit number, the first digit being different from 0.
Two different letters are always coded with two different digits and the same letter is always coded with the same digit.
NOEL's and LEON's codes are both even.
LEON's is divisible by 7 and the number corresponding to the sum LEON + NOEL is divisible by 13.
What is LEON's code?

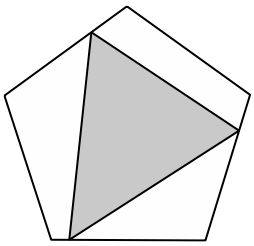
END for C2 PARTICIPANTS

15. This Year's Stars (coefficient 15)
From a convex regular enneagon (9-sided polygon), we can form only two different regular stellated enneagons, by connecting every second (or seventh) vertex, or every fourth (or fifth). If we connect every third (or sixth) we would not obtain a stellated polygon, but rather three separate triangles.



How many different regular stellated 2022-gons can be constructed from a regular convex polygon with 2022 sides?

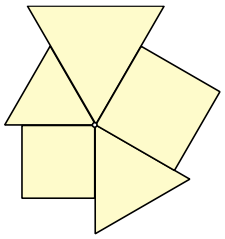
16. A Triangle in a Pentagon (coef. 16)
We choose three points on the circumference of a regular convex pentagon, then connect them to form a triangle.



What is, at most, the ratio of the area of the triangle to that of the pentagon?
Give the answer as a decimal to the nearest 0.001; if necessary, take 2.236 as √5.

END for L1, GP PARTICIPANTS

17. Matilda's Polygons (coefficient 17)
Around a point on his sheet of paper, Matthew has pasted five regular polygons having a common vertex, without leaving a gap around this point and without overlapping any polygons.



Matilda succeeded in the same task with only three regular convex polygons all having different numbers of sides.
What is the sum of the numbers of sides of the polygons used by Matilda?
In Matthew's example where the numbers of sides of the polygons are not all different, the answer would be $3 \times 3 + 2 \times 4$, or 17.

18. Integral Triangle (coefficient 18)
The three sides of a triangular plot are whole numbers of decametres such that the perimeter of the plot expressed in decametres is a number equal to that of its area expressed in square decametres.
What is the perimeter of this land?
Give the answer in decametres.

END for L2, HC PARTICIPANTS