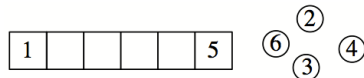# Final FFJM - May 4th, 2013

Information and rankings on http://fsjm.ch

1 – FROM 1 TO 6 (Coefficient 1)

**Place the numbers 2, 3, 4 and 6 in the 4 empty boxes** so that if two numbers, including 1 and 5, are neighbours:
- either they follow each other (for example 3 and 4, or 6 and 5).
- or one is double the other.

2 – DICE OF THE YEAR (Coefficient 2)

Four identical dice have the same dots, that is: 0 dot, 1 dot, 2 dots, 3, 4, and 5 dots. On these dice, two opposite sides always have numbers that add up to 5. Mathilda reads the number 2013 on the four dices in front of her.
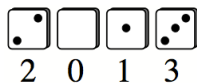
What number does Mathias read when he's sitting opposite Mathilda and can read the opposite side of the dice from what she can?

3 – DIFFERENCE OF THE YEAR (Coefficient 3)

Mathias writes all the 4-digit numbers that use 2, 0, 1 and 3. Then he subtracts the lowest number from the highest number.

What result does he get ?
Be careful: no 4-digit number begins with a zero!

4 – THE FIVE PIECES (Coefficient 4)

Each piece has a white side and a grey side. Mathilda wants to fit all the pieces of the puzzle into the box with the grey side facing up. One piece is already in its place.
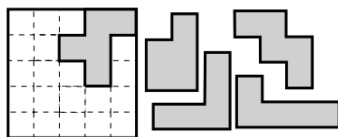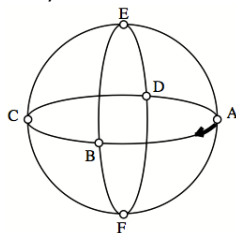
You must position the others, without overlapping.
Draw the outline of each piece in the puzzle with bold lines.

5 – A TRIP TO MAKE YOU DIZZY (Coefficient 5)

On this planet, there are six towns, each one with its own airport and located as in the drawing. Each airport is located exactly 5000 kilometres from its neighbour.

Mr. Tournenvol is an eccentric millionaire who lives in his private plane. He flies from A to B, then turns to the right and travels towards E where he makes a quarter turn to the left. Each time he arrives over a town, he makes a quarter turn, alternating right turns and left turns.

How many kilometres will he have travelled when he returns to A for the first time?

6 – DOCTOR PATRIX (Coefficient 6)

When we write Doctor Patrix's birthday in the format dd/mm (2 digits for the day and 2 for the month – today we are 04/05), we only need the digits 0, 1, 2 and 3.
How many dates could there be with these conditions ?

7 – 21 CUBES (Coefficient 7)

Mathias wants to cut this wooden block into 21 identical little cubes. He uses a very precise electrical saw.
How many straight cuts does he have to make, at least, to obtain his 21 little cubes, given that he can rearrange the pieces in between each cut?
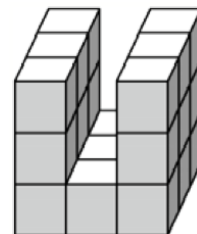
8 – IN THE MIRROR (Coefficient 8)

A three-digit number (greater than zero) is written on Mathias's t-shirt. The numbers are printed like the model above. When he wears his t-shirt and looks in the mirror, Mathias reads a number equal to 45 times that of the number actually printed on his t-shirt.
What number does Mathias read in the mirror?
Note: T-shirt manufacturers are ignorant and don't know that a number with several digits should not begin with a zero.

Problems 9 to 18: beware! For a problem to be completely solved, you must give the number of solutions, AND give the solution if there is only one, or two solutions if there is more than one. For all problems that may admit more than one solution, there is space for two answers on the answer sheet (but there may still be a unique solution).

9 – BY 11 AND 13 (Coefficient 9)

Mathias's number can be divided by 11. Mathilda's number can be divided by 13. Their sum is equal to 316.
What is Mathias's number?

10 – 1, 2, 3 THAT'S ALL (Coefficient 10)

Mathias has a large number of tokens, each bearing 1, 2 or 3. Using these tokens, he makes a sequence so that all the 2-digit numbers made of 2 tokens placed next to each other are different.
What is the maximum number of tokens that Mathias used?

11 – THE MYSTERIOUS NUMBER (Coefficient 11)

A positive whole number smaller than 2013 is such that:
- if we subtract 6, the result can be divided by 6;
- if we subtract 7, the result can be divided by 7;
- if we subtract 8, the result can be divided by 8;
- if we subtract 9, the result can be divided by 9
What is this number?

12 – THREE TIMES MORE HEADS (Coefficient 12)

Mathilda tosses a euro coin and notes the sequence of "heads" (H) or "tails" (T) that she gets one after another. In the sequence of "H" and "T" that she wrote, the sequences of four consecutive throws are all different and in total there is exactly three times more "heads" than there are "tails".

How many letters (maximum) are there in this sequence of throws?

13 – THE THREE FIELDS (Coefficient 13)

Father Spective owns three square fields. Their sides measure whole numbers in decametres. The sum of the surface of the three fields is 222 dam^2.

How long are the sides of the three fields?

14 – THE THREE CONSECUTIVE NUMBERS (Coefficient 14)

The numbers 2013, 2014 and 2015 all have exactly 8 divisors.

What are the three first consecutive whole numbers that have exactly the same number of divisors?

On the answer sheet, write the smallest of these three whole numbers.

END FOR **C2** PARTICIPANTS

15 – THREE PRIME NUMBERS (Coefficient 15)

The product of three prime numbers is equal to 11 times their sum.

What are these three numbers?

16 – THE TRAPEZE (Coefficient 16)

The base of the trapeze has a length of 11 cm and the three other sides are, respectively, 3 cm, 5 cm and 7 cm long.

What is its height?

The answer will be given in centimeters rounded to the closest hundredth using 1.414 for √2, 1.732 for √3 and 2.236 for √5 if necessary.

END FOR **L1** AND **GP** PARTICIPANTS

17 – THE ELECTRICAL CHAIN (Coefficient 17)

A chain of electrical lights has 17 bulbs. Each bulb can be either on or off. The chain is circular: a bulb is chosen to begin then a signal travels to the next bulbs, always in the same direction. When the signal has passed all the bulbs, it arrives back at the first bulb, and so on.

An operation consists in sending the signal from one bulb to the next:

- if the bulb it leaves is on, then the following bulb changes status.
- if the bulb it leaves is off, then the following bulb does not change status.

Before the first operation, all the bulbs are on.

How many operations does it take, at least, for all the bulbs to be on again?

18 – TRINA'S TABLECLOTHS (Coefficient 18)

Trina's table is an equilateral triangle whose side is exactly 1 meter long.

Trina covers her table entirely with five round cloths that have the same radius.

What is this radius (minimum)?

Round to the nearest millimeter, using 1'732 for √3 if necessary.

END FOR **L2** AND **HC** PARTICIPANTS

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