

36th CHAMPIONSHIP: MATHEMATICAL AND LOGIC GAMES

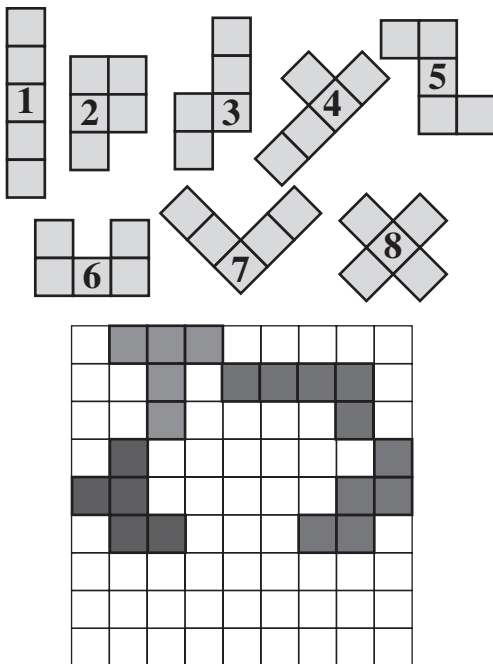
FSJM Quarter finals

Information and results at <http://www.fsjm.ch/>

START FOR ALL PARTICIPANTS

1. THE ENCLOSURE (coefficient 1)

Matilda wants to build an enclosure with a series of pentominoes. She has already placed four on a grid.



The pieces must align with empty squares of the grid, touching one another at a corner, but not at a side, and there must be a closed area inside. Matilda would like to end the enclosure with a single piece from the pieces numbered 1 to 8.

Which piece completes the enclosure so that the enclosed area is as large as possible?

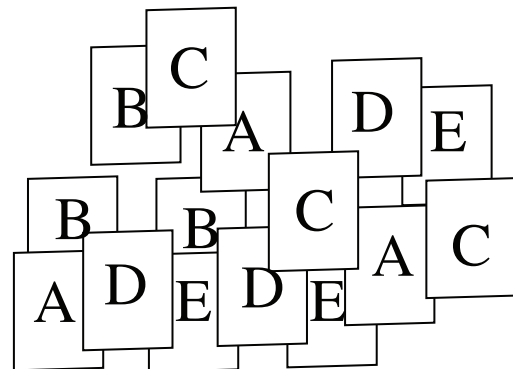
The pieces can be turned and/or placed upside down.

2. THE PIGGYBANK (coefficient 2)

Matilda has 10 2-franc coins and 15 1-franc coins in her piggy bank. She decides to buy a game that costs 17 francs.

In how many different ways can she pay for the game with coins from her piggybank?

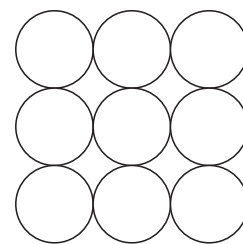
3. GAME OF CARDS (coefficient 3)



Aline is playing a game where you have to eliminate three cards that have the same letter simultaneously, without disturbing any other cards.

In what order could she eliminate this deck of cards?

4. THE STICKERS (coefficient 4)



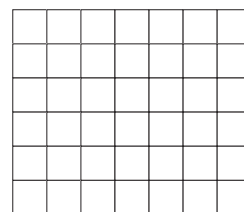
Matthew has 5 red stickers and 4 blue stickers, all round and exactly the size of the disks in the drawing. He must stick each sticker precisely on a white disc

so that:

- no blue sticker touches another blue sticker;
- each red sticker touches at least one other red sticker.

How many red stickers will not touch a blue sticker?

5. THE GRID (coefficient 5)



In this grid of 42 squares, we want to colour squares in such a way that no coloured squares touch one another by a side or by a vertex.

How many squares can we colour, at most?

END FOR CE PARTICIPANTS

6. JOE'S STONES (coefficient 6)

Joe Ellery makes jewellery.

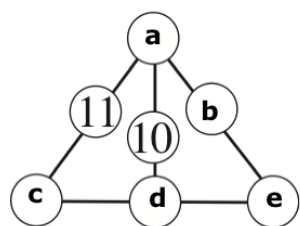
He weighs some precious stones.



Stones of the same shape have the same weight. The three scales indicate 27 g, 24 g and 21 g respectively.

What is the weight in grams of each shape of stone?

7. TRIANGLE OF THE YEAR (coef. 7)



The seven discs in this triangle should contain the numbers 5 through 11 so that the sum of the numbers in each line of three

numbers is 22. The numbers 10 and 11 are already placed.

It's up to you to place the others.

8. COMBINATION LOCK (coefficient 8)

Alex has a combination lock whose 4-digit code, represented as ABCD, is such that:

- A is twice B.
- The sum of B and C is equal to 13.
- The sum of A and B is equal to the sum of C and D.

What is the code for this padlock?

END FOR CM PARTICIPANTS

Problems 9 to 18: beware! For a problem to be completely solved, you must give both the number of solutions, AND give the solution if there is only one, or give any two correct solutions if there are more than one. For all problems that may have more than one solution, there is space for two answers on the answer sheet (but there may still be just one solution).

9. DIGIT-REVERSALS AND SQUARES (coefficient 9)

The number 2022 has a remarkable property: the square of its digit-reversal

is the digit-reversal of its square:

$$2022 \times 2022 = 4,088,484$$

$$\text{and } 2202 \times 2202 = 4,848,804.$$

Matthew found a non-palindromic number smaller than 2022, having the same property, and whose sum of digits is also equal to 6.

Find that number.

10. TWO SUMS & A CAFÉ (coef. 10)

$$\begin{array}{r} a \ b \\ + \ c \\ + \ d \\ \hline = \ e \ f \end{array} \quad \begin{array}{r} f \ e \\ + \ b \\ + \ d \\ \hline = \ c \ a \end{array}$$

In these two additions, only the digits 1 to 6 are used, which are replaced by letters.

The same digit is always replaced by the same letter and the same letter always replaces the same digit.

If $b = 6$, what number does *café* represent?

11. THE LOTTERY (coefficient 11)

In a lottery, one thousand tickets numbered from 000 to 999 were sold. A number is drawn: 205, which wins the jackpot. The five numbers 025, 052, 250, 502 and 520 win a second prize. Finally, all other numbers that have at least two digits in common with the number 205 win a consolation prize.

How many numbers will win a consolation prize?

END FOR C1 PARTICIPANTS

12. JUST ELASTIC BANDS (coef. 12)

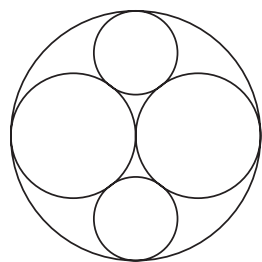
On the label of a box of circular elastic bands it says "weight: 81g; 3 different sizes".

The three perimeters (lengths) of the band types are 12 cm, 20 cm and 25 cm, and the weights of the bands are proportional to their lengths.

In the box, there is the same total weight of each kind of band, each band weighing less than 1 g, but more than 0.3 g.

How many 12 cm bands are there in this box knowing that there are more than 100 rubber bands in total?

13. PIPES (coefficient 13)



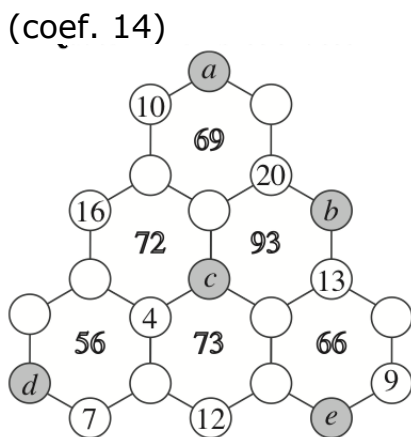
A cylindrical duct contains two large and two small pipes tangential to one another and to the duct, as shown in the figure. The diameter of

each large pipe is 24 cm.

What is the diameter in cm of each of the small pipes?

If necessary, take pi as 3.14. Give the answer rounded to the nearest cm.

14. FOURTEEN DELETED NUMBERS (coef. 14)



Sophie had written the numbers 1 to 22 in the 22 discs in the figure, but Adelaide, her big annoying sister, erased fourteen.

Find the

positions of the numbers erased knowing that:

- each number written in the centre of a hexagon represents the sum numbers placed at the vertices of this hexagon;
- two discs directly connected by a line never contain consecutive numbers.

On the answer sheet, write the values of the numbers a , b , c , d and e .

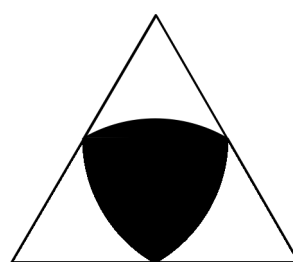
END FOR C2 PARTICIPANTS

15. DOMINIQUE'S AGE (coef. 15)

Dominique's age is a prime number less than 100. If we read this age from right to left, we still have a prime number (which may be the same). If we add all the prime numbers that are strictly less than Dominic's age, we get a number that is divisible by this age.

How old is Dominique?

16. A FISHY PLOT! (coefficient 16)



Mr. Fish has a plot of land in the shape of an equilateral triangle with a central pond in the shape of a curvilinear triangle.

Each curvilinear side of the pond joins two midpoints of sides of the equilateral triangle and is made up of a sixth of a circle with the midpoint of the third side of the triangle as its centre.

Mr. Fish decides to give each of his three children a piece of the plot (each piece in white in the drawing) and to keep only the pond, because he loves fishing above all else.

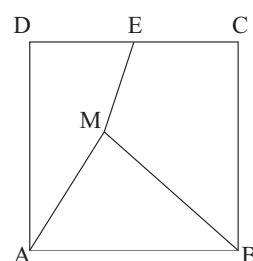
The large equilateral triangle has sides of 100 m.

How much land will each child receive?

Give the answer in m^2 rounded to the nearest m^2 , and if necessary, take $\sqrt{3}$ as 1.732 for and pi as 3.1416.

END FOR L1, GP PARTICIPANTS

17. LOST IN THE FOREST (coef. 17)



Matthew is lost (at point M) in a square forest ABCD (the figure does not reflect his exact position).

His friends Alice, Bertrand and Eleanor

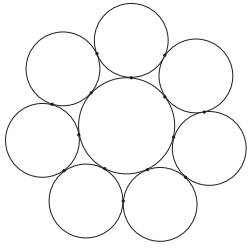
are respectively located at A, B and E, point E being the midpoint of side [DC].

If the side of the forest is 2 km, what is the minimum value of the sum of the distances between Matthew and his three friends?

Give the answer in m rounded to the nearest m. If necessary, take $\sqrt{2}$ as 1.414; $\sqrt{3}$ as 1.732 and $\sqrt{5}$ as 2.236.

18. GRANDFATHER FRANCIS'

ROSETTE (coef. 18)



Grandfather Francis helps his granddaughter make a rosette. They cut out seven identical small discs of radius r and a larger disc of radius R .

They then glue the seven small discs to the large one, each small disc being tangential to its three neighbours. The radius r is a whole number of mm and the radius R is very close to a whole number of mm, the difference in absolute value being less than one thousandth of a mm.

Knowing that r and R are both between 100 and 200 mm, find the value of R .

Give the answer in mm, rounded to the nearest mm. Take $\sin(\pi/7)$ as 0.433884.

END FOR L2, HC PARTICIPANTS